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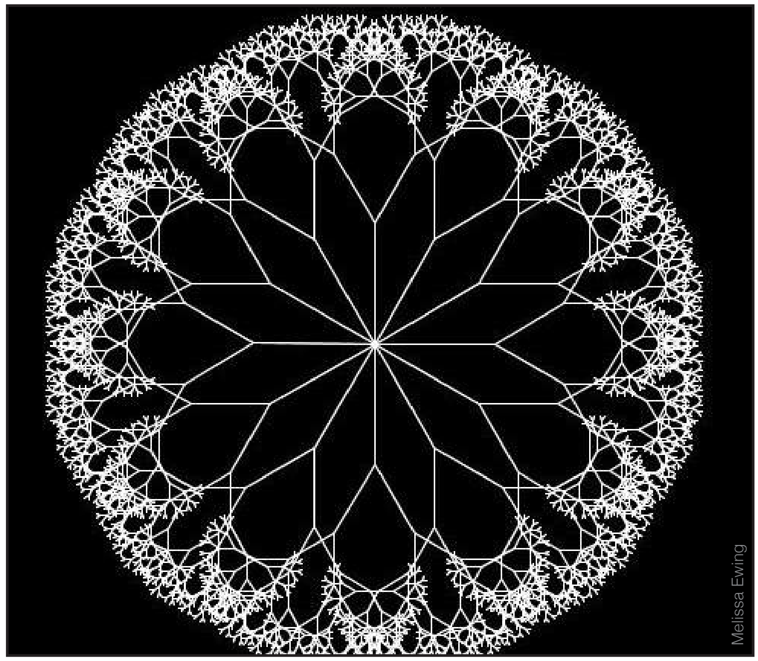


Nature's Algorithms

By Veronica Burnham

Close your eyes.

Think about the veins under your skin. Picture a branching tree stretching towards the sky. Imagine the surface of a pinecone. What can similarities can you observe in the patterns of these images?

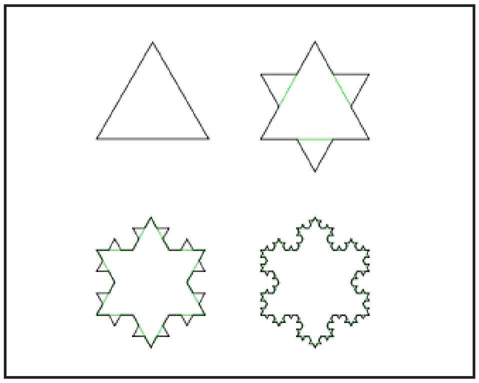
Go back to your mental image of the tree; start at the base of the trunk and work your way up until you reach the first branching point. Follow any one of the branches, and you'll notice that it, too, will eventually split apart. This branching continues until the limbs become too thin to support the weight of the tree's leaves. This sort of pattern is known as a fractal. Pinecones, veins, and tree branches, along with snowflakes and spiraled sea-shells, all feature distinct patterns of conformational repetition.

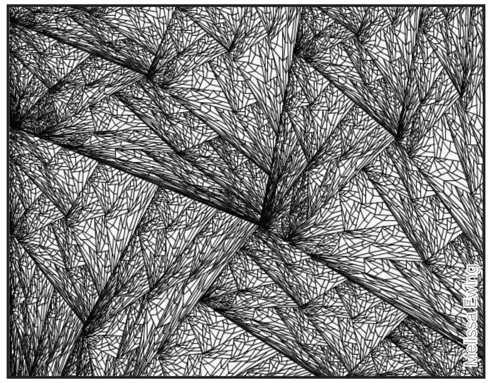
The tree's pattern of branching is recursive; that is, the branching is self-similar and follows the same basic rules at every level as each one becomes smaller and smaller, from trunk to bough to twig. Theoretically, in addition to being recursive and self-similar, fractals are also never-ending. Mandelbrot's set, one of the more popular visual fractals, is a pattern that goes on forever, its stopping point only determined by the artist or programmer's limitations. A video on YouTube, which currently claims to be the deepest zoom yet, magnifies the original image 2.1 x 10^275 times. As the video zooms, you begin to see repetition — within the larger shape are many smaller versions of the whole. Modeled by complex equations, fractals are continuous everywhere, but differentiable nowhere — a fun fact which can probably only be appreciated by someone who has taken a calculus class.

Fractals are not just abstract constructions of the mathematical world; they are found nearly everywhere in nature. Beyond tree branches and veins, fractal-like patterns can be found everywhere from lightning bolts to animal coloration patterns, from heartbeats to some types of broccoli. They are even present in certain animal behaviors. Recently, a study conducted by David Sims of the Marine Biological Association revealed that a number of predatory fish, including multiple types of sharks, use fractals to hunt. More specifically, they use a fractal pattern of movement known as the Lévy flight, in which the animal rapidly changes direction a set number of times, then swims in a straight line for one long period of time, and then returns to moving jerkily. This pattern repeats indefinitely. The movement is self-similar in the sense that the pattern would look the same at any scale, whether the animal is moving through 10 square feet or 1,000. Interestingly, there is a strong correlation between scarcity of food and a predator's adherence to the pattern: less food, more fractals. This suggests that this fractalline system of hunting is programmed into the behavior of these animals, and has arisen because it increases efficiency in hunting — especially in times of prey scarcity when conserving energy is a priority.

Another peculiar aspect of fractals comes from a 1967 study by Benoit Mandelbrot, the mathematician who first coined the term fractal. Mandelbrot created the famous Mandelbrot set (an intricate recursive pattern which is often used in fractal art), and was an integral figure in the early study of fractals. The paper, entitled How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension, tackled this seemingly simple question and got a surprising answer. In his paper, Mandelbrot first explains the fractal qualities of the coastline's shape. First, it is statistically likely that any given part of the coast is a miniature replica of the entire coastline. This is true at any level of magnification. Second, Mandelbrot declared the coastline of Britain to be of infinite length.

The latter sounds like a highly improbable claim. How can a land mass which we can physically observe and measure in its entirety have an infinite perimeter? This strange observation is due in part to the intricate shape of the coastline. Let's say we start by measuring the coast in miles; we have a huge measuring stick that is exactly one mile long. We take that stick and place it end to end all the way around the island. A problem with accuracy immediately arises. If we are measuring with a straight stick, then each mile-long chunk of coast loses all the length it would gain from curvature. As we reduce our measuring stick — a yard, a foot, an inch, one tenth of an inch — our measurements get increasingly more accurate. However, since we





can theoretically shrink our measurement stick infinitely (via division) without ever reaching zero, our measurement of the coast will get infinitely more accurate. Thus, the coastline of Britain is infinite and can never be measured accurately with a measuring stick of any discernible length.

This phenomenon can be similarly observed in a common fractal known as the Koch snowflake. The Koch snowflake begins with an equilateral triangle. For every iteration, each discernable side is divided in thirds. The middle third is taken and copied twice, forming a small equilateral triangle latched onto the side of the original triangle. Now there are three mini triangles budding from each side of the original triangle (see image above for clarification on this process).

At first the image looks like a triangle, starts to become a star, and then begins to resemble a snowflake. With each iteration you increase the length of the perimeter, but once you get to a certain point, this increase is so small it makes no visible change in conformation and no discernible increase in area. With infinite iterations, however, you could magnify the shape continuously, resulting in an endless number of teeny-tiny sides. Thus, just like the coastline of Britain, this shape has an infinite length simply due to the fact that no matter how small of a unit you use to measure it, there will always be a littler one which could measure it more accurately.

Theoretical mathematicians are not the only party who have capitalized on the unique nature of a fractal's perimeter; engineers, too, have exploited the unique utilities of certain fractal patterns. One of the most striking examples of this is the use of fractalization to augment the function of antennas, especially portable ones like those found in cell phones and GPS.

Antennas serve to convert electricity into radio waves and vice versa. They are used to both send and receive radio waves by either radiating electromagnetic waves at a certain frequency or by intercepting these waves. Antennas transmit and receive information for everything from radios to televisions, wifi devices to cell phones. Each of these devices operates at a different frequency, which must be reflected in the length of each antenna. For example, a typical radio receiving both AM and FM information has two antennas. AM radio waves typically have a frequency of 100 kHz while FM waves vibrate at around 100,000 kHz. Because all radio waves travel at the speed of light, higher frequency indicates shorter wavelength. The length of the antenna you need is thus inversely dependent on the frequency – the lower the frequency of the wave, the larger your antenna would have to be. Typically, the length of the antenna has to be approximately half the wavelength of the radio waves—FM radio waves are typically about ten feet long, so about five feet of antenna would have to be coiled inside the metal sheath you see sticking out of many radios in order for you to listen to WOBC.

This phenomenon becomes a problem when an antenna needs to be extremely compact and when a device needs to receive waves at a number of different frequencies. As was the case with the Koch snowflake, fractal patterns allow shapes to greatly increase their perimeter while only slightly increasing their area. In the antenna world, this allows for tight packing of an extremely long antenna into a very confined space. Additionally, researchers have recently shown that fractal antennas allow for sensitivity to several different frequencies. It's like having multiple antennas all wrapped up into one.

Beyond being practical, efficient, and found nearly ubiquitously throughout our natural world, fractals have even managed to invade the human psyche. In particular, fractals have recently been found to dictate our preferences for certain visual aesthetics. This finding comes from a study conducted by a physicist, Richard Taylor, and was found while he was taking a sabbatical in the mid-1990s to pursue a master's degree in art history. Taylor's focus of study was one which made full use of his unique education – studying the mathematical nature of Jackson Pollack's modernist drip-paintings. He studied over twenty of Pollack's canvases, dating from 1943 to 1952, quantifying their fractal dimension. The fractal dimension is a measure used by mathematicians to determine how strictly a shape or pattern adheres to the classical definition of a fractal. For one-dimensional fractals (2D shapes, i.e. a branching line or Sierpinski triangle), the fractal dimension rating ranges from 0.1 to 0.9, while two-dimensional fractals (shapes in a 3D plane, i.e. tree branches) are rated between 1.1 and 1.9, with a greater number in a given dimension indicating closer adherence to classical fractal qualities. Most fractalline shapes observed in nature are rated between 1.2 and 1.6.

Using this information, Taylor analyzed each canvas at numerous locations and magnifications – ranging from 1/10 of an inch to the whole canvas – to see if fractal patterns were present on multiple scales. Not only did Taylor find fractal dimensions, but his findings also seem to support the idea that Pollack knowingly implemented these self-similar patterns. His earlier works display fractal dimensions similar to shapes found in nature. Jack the Dripper's later paintings, however, involved more intricate patterns which gave rise to even higher fractal dimensions than normally seen in a one-dimensional fraction.

Taylor then took a look at how people reacted to Pollock's work in relation to its fractal qualities. After making a number of mock-Pollocks – some with a fractal pattern, some without – Taylor surveyed 120 people on which paintings they preferred. A whopping 113 out of 120 preferred the paintings made with a fractals in mind. A study later conducted in collaboration with the University of Oregon revealed that people are most aesthetically pleased by fractal images with dimensions between 1.3 and 1.5, nearly the same fractal dimension observed in nature.

At first, the idea that our world is filled with complicated, recursive mathematical shapes does not seem intuitive. However, on closer examination, it doesn't make sense for the world to exist any other way. The principle of Occam's razor states that "simpler explanations are, other things being equal, generally better than more complex ones". This philosophical approach has proven itself veritable time and time again in explaining the innermost workings of our universe. It would seem to fit in perfectly with the world of fractals: incredibly complex visual shapes and behavioral phenomena that can be boiled down to a set of simple rules.