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The Cognitive Significance of Mathematics: What the Educational and Psychological Literature – New and Old – Can Tell Us About the Importance of Learning Math

Haddy Dardir

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1. Compute $10!$, $100!$, and $500!$ (Note how quickly this is accomplished.)
2. Expand the binomial $(a + b)^{40}$. Simplify the expression to check that *Mathematica* did it correctly.
3. Calculate and simplify the derivative of $f(x) = \frac{e^{x^2} + \sin^2 x}{e^{x^2} + \cos^2 x}$. Take care to use the proper syntax (i.e., capitalization, parentheses, brackets, etc.) when entering the function. You might find the various palettes (which can be accessed from the **Palettes** menu) helpful in entering your expression.
4. Calculate and simplify the first and second derivatives of $f(x) = \frac{x^3 + 7x^2 - 12}{2x^4 + 3x^2 + 1}$.
5. (a) Find the (indefinite) integral of $g(x) = e^{3x} \sin 4x$. Find the definite integral between $x = 0$ and $x = \pi/6$.
(b) Without computing either integral, say what you can about the method(s) you would use to do so.
6. Use implicit differentiation to find dy/dx , where $4xy - e^{3y} + y^2 \cos 2x = 0$.

The Cognitive Significance of Mathematics

What the Educational and Psychological Literature — New and Old — Can Tell Us About the Importance of Learning Math

Written by Haddy Dardir
 Illustrated by Natalie Covin

Imagine yourself in middle school again. During lunch, you sit with your friends, some of whom complain about the math test they had just taken. You begin thinking about your recent pre-algebra exam and how inapplicable its material was to your immediate self-interests. It leads you to ponder a particular question about the value of math in your education and long-term career goals. You may have heard many of your peers ask this question multiple times before coming to college: "Why do I need to study math if I am never going to use it in my career?"

The answer to this question lies in the social and behavioral mechanisms underlying our intelligence and cognition. However, before foraying into the psychological literature, one must understand the historical context of mathematics's fundamental human thought process and how it became an essential component of the universal school system.

According to the mathematician Gert Schubring and researcher Alexander P. Karp, the roots of the math curriculum are extensive. It began with professional training in ancient Mesopotamia. During this time, math became part of the general

school curriculum in ancient Greece. It rose to popularity over the following centuries to the point it was taught as basic knowledge in every primary school. Educators later reinforced the emphasis on math worldwide in the 1960s with the New Math Movement, which continued to influence mathematical education.

During the early years of mathematical psychology, Edward L. Thorndike (1874-1949), a famous educational psychologist of the early 20th century, published a 1922 journal article capturing how humans interpret and interact with mathematical equations, aptly titled "The Psychology of the Equation." In the article, Thorndike assesses how students view equations, explaining how they are often given an equation and told to solve it with no context. However, Thorndike later indicates that equations are crucial forms of information organization and can be beneficial to one's life, given that the meaning of their variables and unknowns are understood.

As transferable to understanding the relationships between real-life variables as learning equations may seem, this easy transition from paper to practice may not apply to all mathematical topics. According to a report from University of Georgia mathematics educator Jeremy Kilpatrick (1935-2022),

Thorndike published additional research revealing that after being asked to judge the size of a rectangle, students participating in his study did not showcase significant improvement in evaluating the size of a triangle. The study indicated that the specific cognitive skill of judging the size of one shape does not transfer to one's ability to judge the size of other shapes.

Acknowledging the transfer of cognitive skills was much more limited than teachers initially assumed, Thorndike continued to promote connectionism — the theory that states learning is achieved when a learner establishes a connection between a stimulus and response. In a typical math classroom, this theory would most likely appear as the awarding of higher grades to those who solve problems correctly. Connectionism and behaviorism continued to strongly influence math education for many years during the 20th century. However, several psychologists brought Thorndike's work into question.

One of these psychologists was Charles H. Judd (1873-1946), another prominent educational psychologist who lived during the first half of the 20th century. Thorndike was the main target of criticism for Judd, who believed connectionism inaccurately reduced higher mental processes to simply the sum of simple cognitive processes. Judd believed that Thorndike's work led educators to view math as a collection of specific items to be drilled rather than a gateway into abstract, systematic forms of learning. Judd also believed that while Thorndike posited the transfer of mathematical thinking arose from widely applicable generalizations, transfer instead occurs from higher levels of generalization, in which identical elements connect different situations.

Before foraging into the psychological literature, one must understand the historical context of the fundamental human thought process of mathematics and how it became an essential component of the universal school system.

Another psychologist who questioned Thorndike's theory was William A. Brownell (1895-1977). Brownell viewed the behaviorist model of learning as an empty approach to learning math, inhibiting the ability of students to apply math to novel situations. In his 1944 article "The Progressive Nature of Learning in Mathematics," he equated four educational weaknesses in math education with the connectionist view of learning. These included: attention being directed away from the process of learning and too much attention being given to the "product," the pace of instruction being too rapid for students to soak in the material, educators providing the wrong kind of practice to promote learning, and the assessment of error being superficial rather than beneficial to the progress of students.

Although Thorndike's work may have shown that judging the size of a rectangle does not directly translate to judging the size of a triangle, his findings likely missed a key point regarding the cognitive skills facilitated by learning math: applications can

be broad, and the links between certain mindsets and experiences may not always be immediately apparent.

For example, heuristics is another psychological construct that can be utilized to solve problems in math. Humans use heuristics, or mental shortcuts, all the time, from ignoring unimportant emails to memorizing exam question formats. However, late 20th-century studies from the mathematician Edward Begle and mathematics educator Alan Schoenfeld found that using heuristics in math does little to improve general problem-solving skills — much like heuristics in everyday life. Therefore, not only can math encourage students to avoid the overuse of heuristics in math problems but in real-life situations as well.

Regarding math in education today, the subject can be assigned practical, disciplinary, and cultural values, according to Philipp Legner, founder of mathigon.com. First and foremost, Legner acknowledges that after completing the primary school math curriculum — addition, subtraction, multiplication, division, and so on — students can have difficulty understanding the significance of learning secondary school concepts such as solving quadratic equations, sketching graphs, and trigonometry. However, these concepts can transfer to many domains in practical, everyday life, such as personal finance and news statistics. Proficiency in understanding linear relationships and interpreting percentages can give people the upper edge in these domains. While computers can perform these calculations much faster than most humans, they cannot analyze real-life situations in terms of math and interpret results with as much nuance as humans.

Second, as a result of the problems students are often met with in math, taking their time to solve each question sets students on the path to truly understanding the relationship between variables, disciplining them in the process. Moreover, IQ tests always include mathematical and logical puzzles, indicating the ability of math to test the limits of the human mind.

Lastly, the cultural impact of learning math cannot be understated. In the natural sciences, the laws of the universe are written in the language of mathematics as equations governing every natural process. Computers that are apparent in our smartphones, buildings, and transportation also would not exist without math.

While it is true that certain careers require math much more than others, the value of a mathematics education is high for every student — even for those working in jobs that require little to no calculations. Teachers must give children a reason to learn complex topics early in their academic careers to understand the brain growth and cultural competence associated with a math education. According to educational psychology, studying math can enable students to develop problem-solving skills that can allow them to better detect relationships between variables and patterns in the systems around them. As a result, the ability to do math is an essential part of the human psyche that can lead us to a better world. ● ● ●