Bayesian Methodology As a Tool for Scientific Confirmation

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Introduction

Under what conditions can scientists confirm their hypotheses? Karl Popper, a 20th century philosopher of science, was opposed to the necessity of confirmation as being "an essential part of science".[1] Others have disagreed with Popper, illustrating the need for a theory of scientific confirmation rather than holding scientific inquiry as a practice of falsification. From this desire, theories of confirmation arose, like the hypothetico-deductive model and Bayesianism, as a way to explain how scientific hypotheses are confirmed.

This paper explores the confirmation of hypotheses in science, looking at the positives and negatives of using Bayesian probability as an account which offers an explanation of the method of hypothesis confirmation. I begin with a discussion of Probability Theory, covering two main interpretations: the frequency interpretation of probability and the subjective interpretation of probability. From this, I highlight the subjective interpretation and illustrate how degrees of beliefs can be expressed through mathematics as an assigned probability value, which is ascribed to each belief. In light of this, I will show that Bayes' Theorem can relate conditional probabilities and outside information as a way to alter and improve the probability value of the prior probability. Where this outside information provides a probability value not derived from an individual's degree of confidence but rather from a source distinct from the person.

When discussing probability, individuals tend to appeal to one of the main interpretations of Probability Theory, either the frequentist or subjective interpretation. Individuals rarely utilize probability values derived from methodology outside of the given interpretation they hold, and rather try to define the probability value in a way that fits within their given interpretation. I maintain that based on the mathematics of probability this is not necessary, one can begin with a subjective prior probability and use a frequentist derived conditionalized probability to improve the prior probability. I illustrate how this can be done using an equation derived from Bayes' Theorem. Ultimately, I conclude this section by arguing that through the addition of new information, regardless of how it is derived, Bayes' Theorem illustrates that individuals can have a stronger base for reasoning, which will in turn create a more informed mind to assign probabilities.

Given the mathematics of Bayesian Probability, I move to a discussion regarding how Bayesian methodology can be used for confirmation of hypotheses in science. This account of confirmation will be contrasted with the hypothetico-deductive model. From this contrast I argue that by using Bayesian methodology the main shortcomings of the hypothetico-deductive model can be avoided. I then illustrate an argument against Bayes' theory, given by Peter Godfrey-Smith, which highlights that the subjective prior probabilities can vary greatly between scientists. This difference in the priors will then cause, as he argues, the posterior probabilities to be wildly different, in turn illustrating how this methodology cannot be used as an account of confirmation. I provide reasons against this notion, describing how with enough information the posterior probabilities begin to converge on a similar value.

A second argument against the Bayesian method, described by Colin Howson and Peter Urbach, will argue that a subjective interpretation cannot supply science with the required objectivity to provide the necessary authority of scientific knowledge. I maintain that the notion of objectivity in scientific inquiry described by Howson and Urbach is too strict. I will rather argue that a more flexible notion of objectivity, which more closely resembles a rationality requirement, is more appropriate for scientific inquiry.

From this rationality requirement, I argue that scientists are uniquely positioned to assign probability values because they have a more informed background for assigning subjective probabilities. Ultimately, I conclude that the Bayesian method is useful for the confirmation of scientific hypotheses because it allows for the avoidance of many shortcomings normally associated with confirmation in science, while also providing the rationality requirement and thus maintaining the authority of scientific inquiry.

Probability Theory

First, I begin with an introduction to Probability Theory to establish a baseline for my further discussion of Bayes' Theorem. There are multiple interpretations of Probability Theory and I will be focusing on the frequentist interpretation and the subjective interpretation, as they are the two interpretations necessary for the arguments that follow. The frequentist interpretation of probability posits "that relative frequencies bear an intimate relationship to probability".[2] In other words, the probabilities can be seen as directly connected to the events within the reference class. Due to this connection, the frequency interpretation takes probability as the frequency of an event's occurrence or the rate of a certain outcome of the event.

In the frequency interpretation, one "might identify the probability of 'heads' on a certain coin with the frequency of heads in a suitable sequence of tosses of the coin, divided by the total number of tosses".[3] This example gives a more detailed explanation of what probability means in the finite frequency interpretation, there are frequentists who consider infinite reference classes rather than finite, but this distinction goes beyond the needs of this paper. The example of the coin begins with an individual first observing how many actual outcomes of an event are the desired outcomes. By desired outcome I mean the outcome whose probability you are trying to calculate. From this, an individual would take the number of all the observed desired outcomes and divide it by the number of total observed events. This ratio would show the proportion of the desired event against the total number of events. Proportions can then be seen as the probability of observing the desired event in the future, according to frequency interpretations method. By looking at probability through the lens of the frequency interpretation, we give the same weight to each event regardless of where any single event is located in a string of events. In other words, the specific time value of an event has no bearing on the probability of an event, given that all the relevant conditions remain the same.

The way an event is framed also presents a difficulty to the use of the frequentist interpretation. Take that I want to calculate the probability of me living to the age of 100. The event of me living to 100 must be framed by my living in the 21st century with certain medical advances not available to previous generations, that don't smoke, that I exercise regularly and so on. In light of this, the relative frequency must be revised to the event's reference class, the list of attributes I described. This poses a problem because there would be no single answer given by the frequentist, there would be a probability given living in 21st century, a probability given that I don't smoke, a probability given that I exercise regularly and so on.

The second interpretation of interest is the subjective interpretation of probability; this interpretation will be the focus of this paper as it is the interpretation used in Bayesian Probability. Subjectivist interpretation "treats the probabilities of theories as a property of our attitude towards them; such probabilities are then interpreted, roughly speaking, as measuring degrees of belief".[4] First, I will illustrate what this sentence is trying to articulate when utilizing the term "theories". Through our everyday lives we go into the world around us and have experiences, we have these experiences by perceiving the world and bringing all this information together to create experiences. In some cases, we remember specific experiences and store them in our memory. At a later time, we may then reflect on these memories of our experiences and try to understand different aspects of the specific experiences. Once we gain an understanding of our specific experiences of the world we begin to group similar experiences together and make connections between what we see as interrelated experiences. By assembling these relationships between ideas we are beginning to construct our theories about the universe. When we see and understand connections between events we begin to believe that they are connected in reality and thus we create propositions. This shows what is meant by 'theories' in the above sentence, the beliefs that we have about the world based on our previous experiences.

In light of this, I can look at subjective probability and illustrate what it entails in relation to these propositions about the world. To do this we must return to the first part of the definition, specifically when it is stated that subjective probability “treats the probabilities of theories as a property of our attitude towards them”.[5] This means that a person’s attitude toward a sentence can be considered as a value of how probable that individual takes the sentence to be. This assigning probability can be shown in the following question that a person might ask of himself. How much probability do I assign, do I give to the sentence? In other words, we assign a probability value for a given belief that reflects our attitude toward the belief. This allows us to understand probability as simply measuring the degrees of belief based on our attitudes of the validity of belief. We can better understand this notion if we think of assigning these degrees of belief as assigning degrees of confidence. These degrees of confidence would be a relation of how certain I am that a belief I hold is likely to be. From this attitude about the belief I would be able to assign a specific probability value.

Degrees of confidence can also be ordered or put on a scale of probability based on the number we assign. This scale of degrees of confidence can be illustrated with a simple example of two beliefs. Let’s say that I have a belief that ‘2+2=4’ and also a belief that ‘the planet Venus is bigger than the planet Mars’. Now let’s say I am confident that the sentence ‘2+2=4’ is true due to a set of multiple reasons. My reasoning for the belief stems from some knowledge that I have gained from grade school about the values of integers, as well as my development of additive reasoning skills, allowing me to come to the understanding that these specific values when added together produce the answer of 4. From this reasoning I would then have confidence in the truth of the sentence and in turn I would assign a probability based on this confidence. My degree of confidence in the other belief sentence, ‘the planet Venus is bigger than the planet Mars’, might be based on different set of reasoning. This reasoning may arise from the fact that I have once overheard a conversation between two individuals when one said that ‘the planet Venus is bigger than the planet Mars’. From this reasoning I would have some degree of confidence in the validity of the sentence, which would then allow me to assign a probability value that reflects this degree of confidence.

When assigning probabilities we use a scale from 0 to 1, with 0 being no chance that the sentence reflects the universe and 1 being 100% certainty that the sentence reflects the universe. This means that for our example I may assign to the first belief a probability value of 0.99, which would be written as P('2+2=4') = 0.99. In other words, I assigned a probability to this sentence that reflects the certainty I have in the belief. By my assigning this specific probability, it reflects my relative high degree of confidence in the sentence. Due to my reasoning and understanding of mathematics, I have near total confidence in the sentence '2+2=4' as being true. Now in the case of the other sentence I may assign a probability value of 0.05, which would be written as P('the planet Venus is bigger than the planet Mars') = 0.05. Due to the fact that I assigned this specific probability it reflects the relative level of confidence I have in this belief. Similar to the previous sentence this relative degree of confidence is developed from my reasoning about the sentence. This reasoning is based on the little understanding I have about the relationship between the sizes of planets in our solar system. From this we can look at these two distinct probability values on a scale and see that I assigned a much larger probability to the first sentence than I did to the second sentence. I showed that this assigning of values was due to my underlying degree of confidence in the two distinct beliefs, which more importantly reflects the reasoning I had to believe the sentence. Due to my increased knowledge of mathematics and the relatively easy mathematical equation I could have much greater certainty in believing the sentence '2+2=4'. Conversely my limited knowledge of the two planets closest to earth resulted in me not having a relatively high level of confidence in believing the sentence 'the planet Venus is bigger than the planet Mars'.

Conditional Relationship of Beliefs

How do we then change our degree of confidence in our beliefs so that we can increase the probability value we assign to our beliefs? So far we have discussed subjective probability as it relates to single beliefs, where we treated belief sentences as if they had no necessary connection to any other belief sentences. Now we will discuss the subjective probability of conditional beliefs where we consider our beliefs as having a relationship to our other beliefs. This will demonstrate the critical notion of interrelationship between our beliefs, which I will show affects the probability value of a belief. Conditional beliefs can be understood as "beliefs about what the world would be like given that particular things happen".[6] In other words, our degree of confidence in a belief is influenced by other beliefs that we hold. Due to the influence of other beliefs our understanding of probability no longer has a one variable sentence P(A) but rather a sentence resulting in P(A | B), showing the relationship of the influential belief. This new sentence can be defined as the probability of A given the probability of B, conditional on B. This probability value assigned to the P(A | B) can be expressed by P(A | B) = P(A & B)/P(B). In other words, this equation is stating that the conditional probability of A given B is relative to the proportion of outcomes where B outcomes are also A outcomes.

In the following example we will see how other beliefs influence the degree of probability I assigned to a particular belief sentenc-es. The sentence I will take as my belief will be 'it is going to rain today'; this is our belief A. Let's say I looked at a weather fore-cast and saw a slight chance of rain was predicted for today. I then reason from this experience and recall that weather forecasts are not accurate one hundred percent of the time. From my rea-soning I assign a probability value that reflects my degree of con-fidence. This results in me assigning a value of P(A) = 0.2 to the belief.

I also have a second belief, 'it is cloudy outside'; this will be my belief B. Earlier in the day I looked outside my window and perceived that it appeared to be cloudy. I then presume that my senses are trustworthy and from this, reason that my experience of the sky being cloudy was accurate. I also reason that not much time has passed. Based on this reasoning I have a specific degree of confidence that it is currently cloudy outside so I assign a probability value P(B) = 0.8. Now I must consider the probability that it will both be raining and cloudy, this is our P(A & B). In other words, this means the probability that both of these events will occur together. This value cannot be greater than the probability of P(B) because I remember some moments when it was cloudy outside and not raining. Due to these memories I assign P(A & B) = 0.3 to reflect the confidence I have in A and B occurring simultaneously. From these probabilities we can calculate the conditional probability that it will rain today given that it is cloudy today, expressed as P(A | B). This probability can be calculated by inserting our probability values into this relationship P (A | B) = P(A & B)/P(B). So now we take those values, 0.3 / 0.8, and find that the probability that it will be raining today given that it is cloudy has a value of P(A | B) = 0.375. This is a larger value then the value we originally assigned to the sentence 'it is going to rain today', which we assigned a value of 0.2. The observation of cloudiness led to me raising the probability of A. The increase illustrates that the probability of A is now conditional on B. We can say that there exists a conditional relationship because we found a change in the probability value of A to the probability of A given the probability of B. If the two beliefs were not conditionally related then the two probability values, P(A) and P(A | B), would have been equal to each other. If the probability value for A would have been larger than the probability for A given B, then we could say that B has a negative conditional relationship with the probability of A. This means that B would not have supported A and would have acted against A, resulting in the lowering of the probability value for A. This was not the case however as in our example we found an increase in the probability of A given B verses the probability of A on its own. This increase in the probability values means that B helps to give us greater confidence in A, which in turn increases A's probability. In other words, our future degree of confidence in a prior belief can be increased or decreased based on other beliefs that qualify the prior belief.

Bayesian Probability – The Mathematics

Now that we have an understanding of conditional probabilities we can attempt to decipher Bayes' Theorem. This theorem states "the probability of the hypothesis conditional on the evidence (or the posterior probability of the hypothesis) is equal to the probability of the data conditional on the hypothesis (or likelihood of the hypothesis) times the probability (the so-called prior probability) of the hypothesis, all divided by the probability of the data".[7] Let us analyze this definition using terms similar to those used in the previous section as a way to maintain consistency and better understand the meaning. First the definition is saying that the probability of A is conditional on B. With P(A) also being referred to as the prior probability, meaning the probability value given before taking into account any influence by P(B). At first glance this seems to be exactly the same as the conditional probability we just illustrated, this is partially correct but with a small twist. This small twist is that the theorem takes into account the probability of B as also being conditional on the probability of A. In our previous discussion of conditional probability, P(A) was only influenced by P(B), we did not take P(B) to be influenced by P(A).

This probability value for B given A must be attained from external information; this is due to "the data conditional on the hypothesis".[8] In other words, the P(B | A) is representing a conditional relationship between the two beliefs. This data regarding the relationship must be derived from outside the individual due to the fact that the individual has no knowledge of the relationship between the beliefs. This means that the individual would not be able to assign a probability value based on some degree of confidence in the belief, because they don't have a belief regarding this relationship. The data then must be acquired from the world in a way that is distinct form the individuals assigning of probability values.

We can now illustrate that the full theorem is stating that the P(A | B) = P(B | A) x P(A) / P(B). From the equation we can see that Bayes' Theorem, at its core, relates conditional probabilities with external information. This is shown by the two conditional probabilities, P(A | B) and P(B | A), in the equation and the subsequent probabilities, P(A) and P(B), demonstrating how the conditional probabilities are related. Let us look at an example to better grasp this relation of conditional probabilities.

An example of Bayes' Theorem would be as follows; let's say I am trying to determine if the belief that I have developed melanoma (skin cancer) is influenced by my belief that I have been repeatedly sunburnt. To accomplish this, I will use my belief 'I have melanoma' as belief M. Now the fact that melanoma doesn't run in my family leads me to reason that my chances for having melanoma are quite low. I then have a confidence level in belief M that is based on this reasoning. The degree of confidence

Belief S would be that 'I have been repeatedly sunburnt'. I believe that I have been repeatedly sunburnt due to my memory of many summers working outdoors. Due to this reasoning I have a degree of confidence that allows me to assign a probability value of $P(S) = 0.5$. Now, let us take it hypothetically that I have read medical journals and found that of people who develop melanoma 80% of them had also been repeatedly sunburnt in their lifetimes. This external information gives me the probability value $P(S \mid M) = 0.8$, which means the probability of a person to have been repeatedly sunburnt given that this person has melanoma. From this information and our degree of confidence derived probability values we can find the conditional probability of M given S. The conditional probability being that I have skin cancer given that I have been repeatedly sunburnt. We plug the values into the probability equation $P(M \mid S) = P(S \mid M) \times P(M) / P(S)$ and find that $P(M \mid S) = 0.8 \times 0.1 / 0.5$ meaning that $P(M \mid S) = 0.16$. This is a slight increase from the probability that I assigned to my belief M. This means the probability of S led me to raise the probability of the M. In other words, my belief that I have melanoma is now conditional on my belief that I have been repeatedly sunburnt; given that there is a positive conditional probability that people who have been repeatedly sunburnt have melanoma.

This example shows how Bayes' Theorem can relate conditional probabilities with external information. Let us look at exactly what I mean when I say we used Bayes' Theorem to relate external information to a conditional probability. In our example the value of the conditional probability $P(S \mid M)$ originated from external information that related the two beliefs we were discussing. This was the outside medical journal I referenced that gave us $P(S \mid M) = 0.8$. In other words, we took the statistics of a known relationship and used the value to help inform us about the relationship we were trying to find. After we put this information into our expression of the relationship between me having melanoma given that I have been repeatedly sunburnt it allowed us update the probability of the belief. By updating my belief I was able to increase the probability of the belief, which was illustrated by the value of the calculated conditional probability, $P(M \mid S) = 0.16$ was greater than the assigned probability $P(M) = 0.1$. This updating of probability does not need to result in a positive increase in probability. We can have new information that results in the decreasing in the probability of my belief. This would have been shown by P(M | S) being less than the original P(M). Finally, our updating of beliefs need not be a one-time occurrence either, we can continually use new information to alter the probabilities of our beliefs and change their values. To be clear by updating beliefs we are not increasing their probability of reflecting the world but are rather improving our understanding of the relevant information and from this altering the probabilities we assign to our belief accordingly.

Bayesian Probability – Reliability and Reason

With the addition of new information we can improve our probability values, but how does this affect the believer, the individual holding the belief? I will now discuss what it means for us to be good knowers. Ultimately looking at how our ability to be good knowers can affect the reliability of testimony. Ian Hacking suggests that testimony cannot be the only consideration in our reasoning about a situation because testimony does not take into account the "base rates" involved in events.[9] From this, we can see there are underlying probabilities that the individual sometimes does not take into account when judging if a person is trustworthy. I will use an example to better illustrate what these base rates in Bayes' Theorem actually consists of. Let's say that you are on a farm that has 1 horse and 9 cows. On a rainy day there is an animal that breaks the barn door wide open. A person who saw the event says it was a horse that broke the barn door. This person is then tested under similar rainy conditions and it is determined that this person was able to correctly identify the type of animal, horse or cow, 80% of the time. Based on this information most people would consider this person as relatively trustworthy regarding this type of observation, as this person has a high record of picking the correct animal. From this perceived trustworthiness we would reason to the conclusion that the probability of the animal that broke the door being a horse would be P(H)=0.8 because the person is right 80% of the time. This reasoning that she is trustworthy and that the probability value of P(H)=0.8 is incorrect due to not considering the base rate that is involved in the proba-

bility equation. If we put this into the Bayesian Probability model we can more accurately illustrate this notion.
To explain the influence of the base rate, I will use Bayes' Theorem as it relates two hypotheses that are mutually exclusive. We can consider the above example as having hypotheses that are "mutually exclusive and, exhaustive" because the animal to break the barn door was either a horse or a cow, it could not have been both so it must have been one or the other.[10] For Bayes' Theorem, this means that we will have a slightly altered equation than what was shown before, to represent the two mutually exclusive hypotheses. For the new equation I will use the variables A and ~A to represent the mutually exclusive hypotheses. The variables A and ~A do not pertain to the above example and are used only to build the new equation. This distinction between A and ~A is our base rate or the background information necessary to calculate the probability value for P(A | B), meaning it is crucial to be added into our equation. We can derive the new iteration of the Bayesian equation with the ~A included in the calculation. The equation that we used before was P(A | B) = P(B | A) x P(A) / P(B), which doesn't include our ~A. But when we consider that A and ~A are mutually exclusive we can state that the P(B) = P(A) x P(B | A) + P(~A) x P(B | ~ A). From this we can plug the expression into our equation we have a result of P(A | B) = P(B | A) x P(A) / [P(A) x P(B | A) + P(~A) x P(B | ~A)].

Now we will see how this equation works in the context of the previous example regarding the horse and barn. The hypothesis A we are trying to show is that 'A horse broke the barn door'. This value for the probability that a horse broke the door is P(A) = 0.1, since 10% of the animals who could have knocked down the door were horses. Our second hypothesis would then be that it was a cow that broke the barn door. Since the two hypotheses are mutually exclusive the probability value of P(~A) = 0.9, representing the percentage of potential animals that are cows who could have knocked down the barn door. The probability value that it was a horse given that the individual perceived the correct type of farm animal will be P(B | A) and can be acquired by testing the individual who observed the event. This probability value would be derived from a frequentist interpretation of Probability theory, which relies on a set of observation and creates a ratio to show the proportion of the desired event against the total number of events in the given set. In the example, the testing of the individual who saw the event fulfills this frequentist derivation, the test found the individual was correct 80% of the time, this gives the probability value P(B | A) = 0.8. Finally, our probability value that it was cow given that the individual perceived the correct type of farm animal will be P(B | A) = 0.2. This probability value is derived based on the percentage of the time the observer was incorrect, 20% of the time. We can put these values into Bayes' Theorem and see P(A | B) = (0.8 x 0.1)/[(0.8 x 0.1) + (0.9 x 0.2)] = 0.31. From this probability value of P(A | B) = 0.31 we can say that P(~A | B) = 0.69 because the two hypothesis are mutually exclusive, 1 – 0.31 = 0.69. It is more likely that a cow broke the barn door rather than a horse, 0.69 > 0.31. Due to the fact that these equations utilized probability values derived from two different interpretations of probability theory I maintain that individuals no longer need to interpret probability values using only one interpretation of Probability Theory. It is important to note that the testimony of the individual is relevant to the equation. The input of testimony in the equation illustrates that even with the testimony of what many individuals would consider a credible source there is still a greater probability that the animal to break the barn door was a cow. This ultimately shows that we should not rely solely on the individual, who said it was a horse, due to the baseline information showing there was greater likelihood that it was a cow.

The base rate or relevant background information is key for understanding trustworthiness because when it is not taken into account we rely exclusively on testimony and it is always a mistake to rely solely on testimony, when relevant baseline information is available. This was shown in the example where most individuals would have taken the testimony of the individual who observed the event. From this we can state that we require this base rate to assess our confidence in a belief because having knowledge of background information affects the probability of the belief. Ultimately then if we are to be good knowers then we must take all relative information and apply it to our beliefs as a way to have the most accurate probability. We must see reliability and trustworthiness as a function of the impact a person's relevant level of information has on their corresponding degree of confidence. To understand how relevant background information is required for trustworthiness we must turn to how our degree of confidence in produced.

Finally, I will analyze how applying all relevant information can influence our degree of confidence in our probabilities. I will begin by examining how new information influences our confidence in beliefs as it relates to our feelings of uncertainty about beliefs. This type of understanding regarding Bayes' Theorem and how the theorem relates to background information is shown when James Stone states, "Bayesians consider probability to be a measure of uncertainty regarding their knowledge of the physical world".[11] For Bayesians, their knowledge of the world around them is related to their level of uncertainty or degree of confidence in their beliefs. If we are more certain about our beliefs then the probabilities will reflect this change in degree of confidence. To change our confidence in a belief we need to have relevant information regarding that belief. From this it can be stated that when we have relevant information about the world we can improve our probability values.

This is ultimately how Bayes' Theorem improves confidence of our beliefs, we gain new information about the world, which allows us to reason from the information and build confidence in the belief. When the relevant information reflects the world and our confidence reflects the level of reasoning we have in the belief then the confidence is trustworthy. This will then eliminate an excessive or irrational concern about relying on high confidence probabilities because the confidence is based in reasoning about relative information. From this we must say that a person's testimony with higher amounts of relevant information must be considered more reliable than one without adequate information. This ultimately shows that probability favors the prepared or well-informed mind because those with more information have the ability to have a representative degree of confidence in the belief for which they assign a probability.

Bayesian Probability and Scientific Confirmation

Given the mathematics of Bayesian probability, the methodology can be used in relation to confirmation of hypotheses in scientific inquiry. As was shown, Bayes' Theorem represents the relationship between beliefs by the equation P(A | B) = P(B | A) x P(A) / P(B), but let us look at how this relates to the confirmation of scientific hypotheses. The probability value P(A) is our prior probability, this is the probability that the individual assigns to a hypothesis before taking into account any evidence. The probability value P(B) is value assigned to the information or evidence being used to either support or refute the hypothesis, A. The remaining two probability values are conditional probabilities. The probability value P(A | B) represents the posterior probability, the probability of the hypothesis after the evidence is considered. The probability value P(B | A) is a measurement of "how likely the evidence is given the hypothesis".[12] This conditional probability value denotes the degree to which the hypothesis predicts the outcome given the evidence or information. If the posterior probability, P(A | B), is greater than the prior probability, P(A), then the evidence supports the hypothesis. Correspondingly, if the posterior probability is less than the prior probability then the evidence refutes the hypothesis. Given this, it can be stated that with enough supporting information we can confirm hypotheses in science using Bayesian Probability values.

Confirmation by evidence is incredibly important to science, but scientific inquiry is also particular on the breadth of evidence and types of experimentation needed to confirm a hypothesis. In other words, a single type of evidence found using the one experiment couldn't confirm the totality of a theory. An example of this would be that we couldn't confirm the theory of gravity by simply dropping a pencil over and over again and use this evidence to confirm the hypothesis. Different experiments must be conducted to confirm the hypothesis; this is the reason why measuring the movement of planets is also done to confirm the theory of gravity. The need of multiple types of evidence is expressed in the Bayesian method because there is a diminishing return on confirmation if a single type of evidence is continuously used. I will use an example to illustrate how this shrinking return on conformation is accounted for in the Bayesian model.

Take some hypothesis A, based on our degree of confidence in the hypothesis we assign P(A) = 0.2. Given this relatively low probability value for the hypothesis we would also expect to see a low value for P(B), the probability of the evidence B. From this we assign a P(B) = 0.4. Suppose that the evidence was found to follow from the hypothesis, this would give us a probability value for P(B | A) = 1. If we insert these values into Bayes' Theorem, P(A | B) = P(B | A) x P(A) / P(B), we find that P(A | B) = 1 x 0.2 / 0.4 = 0.5. This shows that the evidence confirms the hypothesis A. The interesting question however is what will happen if we use the evidence, B, in an attempt to confirm the same hypothesis.

If we were to repeat the same experiment to confirm hypothesis A by finding the evidence B, our probability values would be slightly different. Our initial probability value would now be 0.5 because of the previous evidence we found, meaning P(A) = 0.5. We would have the same value for P(B | A) because the fact that the evidence follows from the hypothesis did not change, meaning P(B | A) = 1. The probability value for evidence B changes because we have previously experienced this evidence confirming the hypothesis. By having a previous experience of the evidence being used to support the hypothesis the degree of confidence in the evidence changes. There is now a higher degree of confidence that the evidence B will also support the hypothesis in the future, meaning our P(B) increases. Due to this increased degree of confidence we assign P(B) = 0.8. We can now input these values into Bayes' Theorem to see how the same evidence changes the posterior probability of the hypothesis, P(A | B) = 1 x 0.5 / 0.8 = 0.625. This new posterior probability only slightly increased the probability value from 0.5 to 0.625. In light of this example, it can be seen that "each time the theory is confirmed by that kind of evidence, then the probability expressing the degree of belief that it will do so in the future gradually increases".[13] In other words, when the same evidence is used to support the hypothesis the evidence becomes less influential in the probability equation. Thus the margin between the prior and posterior probabilities of hypothesis A will only get smaller and smaller the more times evidence B is used to support hypothesis A because we will be more and more confident that evidence B will occur. Due to only one type of evidence having a diminishing return on confirmation of a hypothesis we can see that the Bayesian methodology reflects scientific inquiry requiring multiple types of evidence being used to confirm a hypothesis.

The Hypothetico-Deductive Model

In light of how Bayesian probability can be used for confirmation of hypotheses in science, let us examine the classic method for confirmation in the scientific method, the hypothetico -deductive model. In this model "we confirm a scientific hypothesis by deducing from it…an observational prediction that turns out to be true".[14] In other words, scientific inquiry begins with the formulation of a hypothesis; we then conduct observations and experiments to test the hypothesis against the empirical data. This testing of the hypothesis will either verify or disprove the hypothesis, meaning we will then be able to consider the hypothesis to be either confirmed or disconfirmed. From this, we can see this method allows scientists to deduce observations about the world from their hypotheses.

This methodology is not without its shortcomings. There are a few well-known deficiencies for the hypothetico-deductive model for confirmation of hypotheses in science; I will touch on two of these shortcomings. The first being, "it does not take account of alternative hypotheses that might be invoked to explain the same prediction".[15] In other words, this model does not give an explanation for a situation where two different hypotheses can explain the same outcome being observed. For this model each theory is equally valid if it can explain the evidence. This means that this methodology cannot offer any insight in choosing or selecting between two theories. We have no evidential grounds to justify choosing or preferring between two hypotheses that are considered equally valid.

We can turn to ecology for an example of two theories that can equally explain the evidence. Say you are an explorer in the Pacific Ocean and come across an uncharted island, on this island is a small species of bird that bears a resemblance to a species found on the mainland. The mainland is 5,000 miles away from this island, much too far for this small bird species to fly. From this, two hypotheses may arise for how this bird spices found itself inhabiting the island. The first hypothesis would be that a few birds from the mainland were carried to the island because of a massive storm and then reproduced while on the island. This ultimately resulted in the birds populating the island. A second hypothesis would be that over time the island drifted farther and farther from the mainland, due to continental drift, until the bird species could no longer fly back to the mainland and where trapped on the island. These two hypotheses equally explain the phenomenon of finding the small bird species on the uncharted island.

The second deficiency in the hypothetico-deductive method is that the method "makes no reference to the initial plausibility of the hypothesis being evaluated".[16] In other words, this model cannot give us any notion or measure of if the hypothesis has any merit whatsoever before evidence is collected. This would allow any hypothesis to be stated regardless of how implausible the hypothesis may be. We would be forced to run tests to deny the hypothesis instead of focusing on the hypotheses that are more plausible. These two shortcomings illustrate that there is a need for a theory of confirmation in scientific inquiry that can limit the deficits of this classic model.

This deficiency can be shown in the example previously given, regarding how the birds came to inhabit the island. Let us say that a person gives a third hypothesis to explain the phenomenon. The third hypothesis being that the birds are on the island because they built a boat and collectively floated their way to the island. Instead of focusing our resources and attention on the two hypotheses previously given, that the bird's inhabitance of the island was due to a large storm or continental drift, we would be forced to investigate this third implausible hypothesis.

The Positives of Using Bayesian Methodology

As previously argued, the hypothetico-deductive model does not allow for the comparison or choosing between two theories that explain the same observed phenomena. In this model these two hypotheses would be equally viable. However, Bayesian probability can be used to create a "Bayesian algorithm for theory preference".[17] In other words, by modifying the original Bayesian probability equation we can have a value of preference, thus allowing us to compare two hypotheses that explain the same phenomena. This can be derived from the original Bayesian probability equation that was previously given, $P(A \mid B) = P(B \mid A) \times$

(A) / P(B). To derive this algorithm, we must first take the Bayesian probability equation and consider its use for multiple hypotheses. The second hypothesis we will use is a hypothesis contrary to hypothesis A, we will use ~A for this contrary hypothesis. A and ~A are mutually exclusive because they cannot both be true, meaning we can state that the P(B) = P(A) x P(B | A) + P(~A) x P (B | ~ A). From this we can insert the expression into our equation and have a result of P(A | B) = P(B | A) x P(A) / [P(A) x P(B | A) + P(~A) x P(B | ~A)].

From this expression, representing two mutually exclusive hypotheses, we can move to an expression with any number of mutually exclusive hypotheses. This equation would have the sum of all the mutually exclusive hypotheses, located in the denominator, this would appear as follows:

$$P(A_i \mid B) = \frac{P(B \mid A_i) \times P(A_i)}{\sum_{j=1}^{n}[P(A_j) \times P(B \mid A_j)]}$$

. To complete the algorithm for the comparison of two hypotheses we can use the previously stated equation for multiple distinct hypotheses. By using i = 1 and i = 2 for each hypothesis respectively, we can write out the above equation for each of the two hypotheses. This would give us two equations

$$P(A_1 \mid B) = \frac{P(B \mid A_1) \times P(A_1)}{[P(A_1) \times P(B \mid A_1)]+[[P(A_2) \times P(B \mid A_2)]}$$

and

$$P(A_2 \mid B) = \frac{P(B \mid A_2) \times P(A_2)}{[P(A_1) \times P(B \mid A_1)]+[[P(A_2) \times P(B \mid A_2)]}$$

, representing the probability values of the two hypotheses. The denominators of those to equations are exactly the same, because of this when we assign them as a ratio we receive the Bayesian algorithm for theory preference, shown as

$$\frac{P(A_1 \mid B)}{P(A_2 \mid B)} = \frac{P(B \mid A_1) \times P(A_1)}{P(B \mid A_2) \times P(A_2)}$$

. This ratio of the two probability values allows us to make determinations on which of the two hypotheses is preferable given the same data. Observing the value of the ratio can identify the preferred hypothesis, if the ratio is less than 1 we should prefer the hypothesis A_1 and if the ratio is greater than 1 we should prefer hypothesis A_2. This shows the advantage of using the Bayesian Methodology in scientific conformation as we can avoid a major shortcoming of the hypothetico-deductive model, not being able to compare hypotheses that explain the same phenomena.

The second reason why the Bayesian methodology should be used is because the hypothetico-deductive model also has the deficiency of not having any way to measure the plausibility of a hypothesis before any testing is conducted. Plausibility arguments are "designed to answer the question 'is this the kind of hypothesis that is likely to succeed in the scientific situation in which the scientist finds himself or herself?' On the basis of their training and experience, scientists are qualified to make such judgments".[18] In other words, by lacking a plausibility argument in the hypothetico-deductive model, all hypotheses would require testing to determine their validity. The Bayesian methodology works differently and in fact has a plausibility argument for limiting irrational hypotheses. The subjective prior probability value, based on a scientist's degree of confidence in a hypothesis, can act as a gage for plausibility. When a scientist has a low degree of confidence in a hypothesis they would assign a prior probability value that is reflective of this degree of confidence. This value means that we can then use the expertise of scientists to limit or disregard those hypotheses with low prior probability values. This limiting will allow us to test only the hypotheses that experts feel are plausible and thus are worth taking the time to formulate experiments. By accounting for prior probabilities the Bayesian model has a way to limit the irrational hypotheses and is thus a superior model for scientific inquiry.

Critique – Subjective Prior Probabilities

Given how Bayes' theorem can be utilized in scientific confirmation and can be viewed as preferential to a classic model for scientific confirmation, I will begin to analyze the critiques of using Bayesian methodology in scientific inquiry. The first critique stems from the subjectivity of the prior probability values, these are the initial values assigned to the hypothesis by the individual, as previously shown. The subjectivity of priors "can seem to be a weakness because Bayesianism cannot criticize very strange initial assessments of probability. And, one might think, where you end up after updating your probabilities must depend on where you start".[19] In other words, there can be a large variation in the way individuals assign the prior probability value of a hypothesis, based on their individual degree of confidence in the hypothesis. It can then be argued that because the prior probabilities may be drastically different the subsequent posterior probabilities, the probability values after evidence is considered, will also be drastically different.

I would argue against the notion that the subjectivity of prior probabilities can lead to drastically different posterior probabilities. No matter how divergent the prior probabilities regarding a hypothesis are scientists are highly likely to converge on a similar probability value given enough information. This can be seen in the previous example regarding why a bird species in currently inhabiting an island far from the mainland.

Let us say that two scientists are in disagreement regarding the first hypotheses, that a few birds from the mainland were carried to the island because of a massive storm and then reproduced while on the island, and its ability to predict why the birds are now on the island. For simplicity I will use the letter A to represent the hypothesis and the letter B to represent the phenomena of finding the birds 5,000 miles from the mainland. In the example scientist one, assigns a relatively low probability value to hypothesis A, $P(A) = 0.2$. The assigning of this low probability is due to scientist one's reasoning that a storm could not carry the small birds the needed 5,000 miles to reach the island. Based on the scientist's degree of confidence, scientist two assigns a different probability value to hypothesis A, $P(A) = 0.6$. This is a relatively higher probability value than the value assigned by scientist one and reflects scientist two's reasoning that a storm a sufficient size could carry the small birds 5,000 miles from the mainland.

Suppose that a case was recently discovered where birds were swept up into a hurricane-sized storm and were flung 6,000 miles from their original location. This means that B has been confirmed through an experiment to follow from hypothesis A. Each of the two scientists would then need to put these probability values into the Bayesian model. Their previous probabilities would then be altered by a factor of $P(B \mid A) / P(B)$. Given that B was confirmed experimentally to follow from hypothesis A, then $P(B \mid A) = 1$. We can then insert this value into the equation and see that the prior probability is now altered by a factor of $1/P(B)$. $P(B)$, the probability value assigned to the evidence B when the validity of the hypothesis is not assumed, must then be set by the two scientists. Scientist two, who assigned a relatively high probability to A, will see B as more likely to occurred and will thus assign a probability value that is relatively higher, P(B) = 0.8. Scientist one, who assigned relatively low probability to A, will see P(B) it be more unlikely and will then assign a probability value that is relatively low, P(B) = 0.3. When these values are then entered into the Bayesian model we find that P(A | B) for scientist one equals 1 x 0.2 / 0.3 = 0.66 and the P(A | B) for scientist two equals 1 x 0.6 / 0.8 = 0.75. The posterior probability values for the hypothesis are much closer for the two scientists, 0.66 and 0.75 respectively, than the prior probabilities, 0.2 and 0.6 respectively. This shows that as more evidence supporting hypothesis A comes to light, and is available to both scientists, they will converge on a similar posterior probability value for the hypothesis.

Critique – Science as Objective Inquiry

One critique of the Bayesian methodology touched on by Colin Howson and Peter Urbach arises from the perceived requirement for objectivity in science as it relates to the subjectivity of the prior probabilities in the Bayesian method. It is argued that objectivity is "ideal for scientific inquiry, as a good reason for valuing scientific knowledge, and as the basis of the authority of science in society".[20] In other words, maintaining objectivity in scientific inquiry allows for the authority of scientific knowledge in our society. This allows people without an understanding of science or scientific inquiry to trust the claims and theories of scientists. Objectivity is science "contends that science should be value-free and that scientific claims or practices are objective to the extent that they are free of moral, political and social values".[21] This requirement of objectivity, that no values should be used in science, is extended to all aspects of scientific inquiry. Under this notion of objectivity no values can be used in the making of a hypothesis or the gathering of evidence and so on.

Colin Howson and Peter Urbach display an argument, against the Bayesian methodology, from "an influential school of thought, which denies that there should be any subjective element in theory-appraisal at all".[22] This school of thought believes that subjectivity has no place in science. The desire for objectivity in science is exemplified when Sir Ronald Fisher, who ascribes to this school of thought, states, “as measuring merely psychological tendencies, theorems respecting which are useless for scientific purposes”.[23] This shows how strongly these individuals feel regarding the use of subjective degrees of confidence in assigning probabilities. These individuals feel that “science is objective to the extent that the procedures of inference in science are”.[24] This is not the case in the Bayesian methodology as the prior probabilities reflect the personal degree of confidence, and the procedures of inference are constrained only by Bayes’ Theorem.

Due to the subjectivity of the priors, there is the fear that a lack of consistency may arise from this methodology. Edwin Thompson Jaynes, who also an ascribes to this school of thought, goes on to claim, “the most elementary requirement of consistency demands that two persons with the same relevant prior information should assign the same prior probabilities…objectivity requires that a statistical analysis should make use, not of anybody’s personal opinions, but rather the specific factual data on which those opinions are based”.[25] If individuals ascribing the prior probabilities were assigning based exclusively on the relevant information then there would be absolute consistency with regards to the values assigned to the prior probabilities of the same hypothesis. This is not the case however as individuals are rarely in agreement on the prior probability value assigned to a given hypothesis.

The lack of consistence must then be questioned. Why are the individuals assigning different prior probabilities when they have the same relevant information? The prior probability values are formed by the degree of confidence of the scientist, so it is possible that in some cases the degree of confidence is impacted by the beliefs and values of the scientist. In light of this, if the beliefs of the individual scientists are used for the formation of probability values there is a concern that the input of beliefs on the part of the scientists will bias the prior probability values and cause the inconsistency we see. If scientist’s can skew the prior probability values with the input of their own beliefs then the Bayesian method does not satisfy this value-free objectivity.

I, however, argue against this need for value-free objectivity in science because the need of value-free objectivity is too high a standard for scientific inquiry. Scientific inquiry should rather strive for a less strict objectivity, something closer to rationality. While value-free science is important when gathering data and accepting which hypothesis accurately explain the world, it is not necessary when deciding which topics or hypothesis to research, as is done with the prior probability values.

If we regard science as a value-free objective inquiry when crafting hypothesis then we are, in a sense, taking the scientists out of science. We would be classifying science as something that any individual could do regardless of background, because the individual beliefs of scientists are not a part of the process. Scientific inquiry is not a static pursuit, done without the subjective individual, it is an active pursuit where scientists input their ideas and values based on what they have learned and experienced. Instead of demanding value-free objectivity in science we should demand "that the investigator make every effort to bring all of his or her relevant experience in evaluating hypotheses".[26] In other words, when crafting hypothesis and assigning prior probabilities values to them, scientist should use their relevant experience to make decisions. Instead of the subjectivity of the priors being seen as a negative it should be seen as a positive. We should allow scientists to use what they have learned to influence their reasoning in regards to their degrees of confidence and assigning of probability values.

This call to have rationality or to only use relevant experiences when evaluating a hypothesis can also reduce the fear of basis in science. When only using relevant information a scientist will not bring in personal biases and will rather only utilize experiences of dealing with similar problems. These relative experiences will then help the scientist to preform science in a way that reflects what he or she has learned. Allowing scientists to use their experiences will then benefit future scientific pursuits as individuals learn from past mistakes. In light of this, Bayesian methodology not only improves on the shortcomings of the classical method but also fulfills a needed requirement of rationality used in scientific inquiry.

Conclusion

I opened this paper with a discussion of Probability Theory, illustrating the two interpretations, those being frequentist probability and subjective probability. From this, I went on to discuss how subjective probability illustrates how probability can be seen as an individual's degree of confidence in a belief sentence. In light of this, I showed that we have beliefs that are conditionally related to each other. In other words, our degree of confidence in a belief can be influenced by our degree of confidence in other related belief. From this understanding, I described the mathematics behind Bayesian Probability, demonstrating that Bayes' Theorem relates conditional probabilities with external information. The mathematics allowed me to show how new information allows us to influence the probability of a given belief. In light of this, I demonstrated how a frequentist derived probability value could be used to improve a subjectivist probability value. Due to these two interpretations working together in Bayes' Theorem, I maintain that we no longer need to interpret probability values using only one interpretation of Probability Theory. Ultimately, I concluded this discussion by showing how human reasoning improves due to the use of new information and from this builds a more prepared mind to assign probabilities.

Given this, I moved to a discussion regarding Bayesian probability as it relates to scientific confirmation, looking at both the strengths and weaknesses of using the Bayesian methodology. I first outlined the classical methodology used for confirmation of scientific theories, the hypothetico-deductive model. From this I illustrated the shortcomings of the model, highlighting three major issues. Given these issues, I highlighted how Bayesian methodology can be used as a way to confirm hypotheses in science, while also bypassing the problems associated with the hypothetico-deductive model. I then articulated a critique of the Bayesian methodology, which states that because prior probabilities are subjective two scientists can have drastically different prior probabilities for the same hypothesis and that this prior bias will affect the posterior probabilities as well. In light of this critique, I argue that the input of subsequent evidence should, in the case of a scientist with definite values but an open mind, lead to correction of unwarranted bias.

Another critique of using the Bayesian method was then analyzed and in doing so displayed how a subjective probability theory would not supply the necessary objectivity that is seen as the ideal for scientific inquiry. From this critique I argued that the need of objectivity is too high a standard for scientific inquiry and that we rather should strive for rationality. Using Bayesian probability was then shown to have no issues supplying this newly required rationality for scientific inquiry. Ultimately, I concluded that Bayesian methodology is a useful tool for scientific confirmation because it lacks the shortcomings of the classic model and also fits the rationality requirement for the scientific methodology.

Notes

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22. Howson and Urbach, "Scientific Reasoning: The Bayesian Approach.", 11.
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24. Ibid., 413.
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