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Unwinding the Web: The Mathematical Basis of the Spider Web

Victoria Fisher

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The Mathematical Basis of the Spider Web

Written by Victoria Fisher
Illustrated by Delaney McRitchie *(Top)* and Claire Segura *(Right)*

Imagine you are walking through a dewy forest one morning. As you pass below the trees you suddenly crash into a spider web, which is perched between two branches. Many of us would scream for fear that one of those small, scary arachnids would end up in our hair, on our clothes, or elsewhere on our bodies. For us, this may be a frightening experience, but for the spider, they may have just lost their home or hunting grounds, an integral component to their existence. While most spiders build webs, the styles and structures of the webs are as diverse as spiders themselves. There are over 45 thousand different species!

The many unique web types are all developed with specific uses in mind. Take, for instance, the "triangle web," which is created by spiders who do not release venom. This web is a horizontal triangle made with fuzzy silk, that spiders can catch and smother their prey on. Another type of web that you may have found in your home is the "tangled web." This is a three dimensional and messy web used to trap the spider's prey. While the webs typically found in homes are created by only one or a few spiders, up to thousands of spiders will work together to create a massive multilayered net—like we see in the scene with Aragog from Harry Potter and the Chamber of Secrets. However, some webs are not used for catching prey at all. Consider the "funnel web," which, as the name describes, is a large funnel of different silk threads. These can be used to store eggs, to hide from predators, or as homes for males as they wait to mate with a female.

The "orb-style" web is the most familiar structure, as it is one of the most common types of webs for spiders in North America. The orb-style web is a flat, two-dimensional net that has been the focus of computational and mathematical research in webs. They are made out of very strong silk produced by glands in spiders' abdomens. While spiders vary in the exact types of silk they make, the proteins that make up the silk contain three different substances: pyrolidin, potassium hydrogen phosphate, and potassium nitrate. These strengthen the silk against environmental stressors such as changes in pH, bacteria and fungal growth, and changes in humidity. They help create a very strong and elastic material that maintains the structure and formation of the web, even when it is stretched by 30–40 percent.

When a spider begins creating an orb web, it will place itself on a secure surface, such as a tree branch or beam. The spider will place what is called attachment silk, which is made in the pyriform glands, on this surface. The spider will then wait for a gust of wind to blow it across to another secure platform where it will attach this first strand. However, before the flight, it must first calculate precisely how far it will have to move so that it can create the appropriate amount of silk. Assuming the strand it has produced is satisfactory, the spider will then carefully walk back and forth along this tightrope-like line, adding more silk to strengthen the thread. The spider creates an additional strand that is centered on and perpendicular to the initial threads, making a Y formation. The spider produces seventeen more radii, which connect to the center of Y, but it will only use seven to create the final structure.

The next step for the spider is to create a logarithmic spiral, which is defined by the following equation:

$$r = ab^{e\theta}$$

A and b are arbitrary variables and theta is the angle from the origin or center. A logarithmic spiral is sometimes referred to as the golden spiral, given its relationship to Fibonacci numbers and the golden ratio. A logarithmic spiral is formed by drawing segments between equally spaced rays, where the segment is perpendicular to the ray to which it is connected. The logarithmic spiral is created by the spider starting from the center of all the radii and then proceeding outward. With only seven different strands, the spiral will grow rapidly, forming large gaps in the web, making it very ineffective at catching prey. Thus, the spider uses a non-sticky kind of silk called "walking thread," which is produced by the major and minor ampulleceae glands; the minor type produces silk that is only half as thick and strong as the major gland. Since the silk used is not sticky and the spiral creates large gaps in the web, this spiral will not be present in the final shape. However, the logarithmic spiral provides a baseline shape for the spider to follow to complete its web.

The next (and final) step for the development of the orb-style web is the creation of an arithmetic spiral. The arithmetic spiral is defined by the equation:

$$r = a\theta^{1/n}$$

A is an arbitrary variable, theta is the angle from the origin or center, and n is the factor that determines how tightly the spiral (or web) is bound. The arithmetic spiral is much tighter than the previous logarithmic structure, and it forms the shape typically seen in a spider's web. In order to help form this spiral, the spider follows the line of the logarithmic spiral, eating the walking thread as it goes along. However, the spider must also calculate precise curvature needed for the new spiral, which requires a much more complicated formula:

$$\text{arc curvature} = \frac{|n|\theta^{1-1/n}(1+n+n^2\theta^2)}{a(1+n^2+\theta^2)^{3/2}}$$

This process is entirely automatic in the spider, demonstrating just how impressive their innate skills are.

As the spider makes the new spiral, it deposits adhesive droplets along the silk. The adhesive proteins are created by the aggregate gland, while the silk itself is made by the flagelliform glands. These droplets are necessary to give the silk its sticky texture so that unsuspecting insects are trapped when they come into contact with the web. The web is also very difficult to see. The silk is very thin with a diameter of around 0.15 millimeters, further, the precision with which the spider creates the arithmetic spiral causes the web to be near invisible. In fact, in its initial state, the web is imperceivable to the human eye. However, if light is reflected off the web at a specific angle or water droplets fall on the strands, altering their orientation and shape, the web may become visible.

The creation of a web is a very intricate and resource heavy process for these tiny creatures. Even after the spider has created one web, it will begin this sequence all over again the following day or night, as the web is rendered unusable within a day of its creation. While some spiders will recycle the proteins needed for the web by consuming the old one, you can imagine much of the spider's energy goes into creating all the materials for these massive and sticky nets. The spider must also take into account different environmental factors such as weather and prey type. This results in each spider having a highly specialized method for creating their webs and capturing their prey. So, next time we take a walk through the woods, we should be careful where we step—we might save one of these mathematical wonders. • • •